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NNLO massive corrections to Bhabha scattering and theoretical precision of BabaYaga@NLO☆

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Abstract

We provide an exact calculation of next-to-next-to-leading order (NNLO) massive corrections to Bhabha scattering in QED, relevant for precision luminosity monitoring at meson factories. Using realistic reference event selections, exact numerical results for leptonic and hadronic corrections are given and compared with the corresponding approximate predictions of the event generator BabaYaga@NLO. It is shown that the NNLO massive corrections are necessary for luminosity measurements with per mille precision. At the same time they are found to be well accounted for in the generator at an accuracy level below the one per mille. An update of the total theoretical precision of BabaYaga@NLO is presented and possible directions for a further error reduction are sketched.

Keywords: meson factories, luminosity, Quantum Electrodynamics, radiative corrections, Monte Carlo
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1. INTRODUCTION

Because of its experimental and theoretical characteristics, Bhabha scattering is the prime process used at e^+e^- colliders to monitor their luminosity [1]. At the GeV-scale e^+e^- colliders, from ϕ to B factories, large angle Bhabha scattering is measured with experimental uncertainties at the per mille level. Correspondingly, the Monte Carlo (MC) generators used in the data analysis, like BabaYaga@NLO [2, 3, 4], BHWIDE [5] and MCGPJ [6], are precision tools that include the large logarithmic contributions due to soft and collinear multiple photon emission matched with the exact next-to-leading order (NLO) photonic corrections. In addition to the these ingredients, the MC codes include the effect of leptonic and hadronic vacuum polarisation, the latter computed using a data based routine for the evaluation of the non-perturbative light quark contribution [7].

The above framework implies that the MC programs are affected by a theoretical uncertainty stemming mainly from missing (subleading) contributions at the level of NNLO radiative corrections. Therefore exact NNLO calculations are the benchmark for a reliable assessment of the error associated to the MC predictions. Since the whole class of the NNLO QED corrections to Bhabha scattering became recently available (see [1] for a review), various comparisons between the

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exact calculations and the MC results allowed to conclude that the theoretical accuracy of the most precise luminosity tools is at the level of one per mille. Although this uncertainty seems sufficient by comparison with the present experimental error, there is still room for a more reliable assessment and, presumably, a further error reduction.

Here we focus on the NNLO massive corrections to Bhabha scattering, for which only preliminary results limited to leptonic corrections for a few experimental selections were used in the most recent, official estimate of the MC precision [1]. These limitations are overcome in the present study [8], wherein the full set of NNLO leptonic and hadronic corrections is computed and all the event selections of experimental interest at meson factories are addressed. The exact NNLO results are compared with the corresponding $O(\alpha^2)$ truncated predictions of the MC BabaYaga@NLO.

2. THE NNLO LEPTONIC AND HADRONIC CORRECTIONS

The full treatment of $N_f = 1, 2$ massive corrections to Bhabha scattering requires the calculation of *i)* pure two-loop diagrams, including irreducible vertex and box corrections that are known to give rise to large leading $L^3, L = \ln(s/m_f^2)$, collinear contributions *ii)* loop-by-loop corrections *iii)* real photon corrections and *iv)* corrections due to real lepton and hadron pair emission. The latter generate L^3 contributions canceling those from irreducible virtual loops.

2.1. Exact calculation

Exact electron loop corrections ($N_f = 1$) were first computed in [9, 10], while heavy fermion and hadronic loop contributions ($N_f = 2$) were later calculated in [11, 12, 13, 14, 15]. As said above, these virtual contributions must be combined with the real corrections. The cross section with complete NNLO leptonic and hadronic corrections can be written as

$$\frac{d\sigma^{\rm NNLO}_{N_f=1,2}}{d\Omega} = \frac{d\sigma^{\rm NNLO}_{\rm virt}}{d\Omega} + \frac{d\sigma^{\rm NLO}_{\gamma,\rm soft}(\omega)}{d\Omega} + \frac{d\sigma^{\rm NLO}_{\gamma,\rm hard}(\omega)}{d\Omega} + \frac{d\sigma^{\rm LO}_{\rm real}}{d\Omega} \qquad (1)$$

which can be rearranged as

$$\frac{d\sigma^{\rm NNLO}_{N_f=1,2}}{d\Omega} \equiv \frac{d\sigma^{\rm NNLO}_{\rm v+s}(\omega)}{d\Omega} + \frac{d\sigma^{\rm NLO}_{\gamma,\rm hard}(\omega)}{d\Omega} + \frac{d\sigma^{\rm LO}_{\rm real}}{d\Omega} \qquad (2)$$

where ω is a soft-hard photon separator. The terms entering Eq. (2) have been computed using various semi-analytical and MC tools developed over the years, *i.e.* by the package bha_nnlo_hf [16] for the virtual + soft corrections $d\sigma^{\rm NNLO}_{\rm v+s}(\omega)/d\Omega$, the MC code BHAGEN-1PH [17, 18] for hard photon radiation $d\sigma^{\rm NLO}_{\gamma,\rm hard}(\omega)/d\Omega$, and the generators HELAC-PHEGAS [19] and EKHARA [20, 21] for real lepton and hadron pair emission $d\sigma^{\rm LO}_{\rm real}/d\Omega$, respectively. For the R contribution to the hadronic vacuum polarisation we adopt the recent compilation encoded in the routine VP_HLMNT_v2_0 [22, 23, 24]. Strictly speaking, EKHARA allows the computation of real pion pair emission only and no other generator exists for the calculation of similar processes containing other hadron pairs. However, we observed in our numerical study [8] that the cross section for real pion pair emission is always very small at all meson factories (at the level of 10^{-5} times the Born cross section or below it). Therefore an educated guess is that the processes of pair emission containing hadrons heavier than pions have a negligible effect in our numerical predictions.

2.2. Exact numerical results

In Table 1 we show the relative contribution of the exact NNLO massive corrections for all the interesting experiments at meson factories and using realistic reference event selections used in luminosity measurements. In Table 1 $S_x = \sigma_x^{\rm NNLO}/\sigma_{\rm BY}$ with $x = e^+e^-, lep, had, tot$, where *tot* stands for the sum of leptonic (*lep*) and hadronic (*had*) corrections, and $\sigma_{\rm BY}$ is the full cross section from BabaYaga@NLO. We refer to [8] for details about the cuts used in our analysis.

Some comments are in order here.

Table 1: Exact relative NNLO massive corrections $S_x = \sigma_x^{\rm NNLO}/\sigma_{\rm BY}$, in per mille.

	$\sqrt{s}$	$S_{e^+e^-}$	S_{lep}	S_{had}	S_{tot}
KLOE	1.020	-3.935(5)	-4.472(5)	1.02(4)	-3.45(4)
BES	3.097	-2.246(8)	-2.771(8)	–	–
BES	3.650	-1.469(9)	-1.913(9)	-1.3(1)	-3.2(1)
BES	3.686	-1.435(8)	-1.873(8)	–	–
BaBar	10.56	-1.48(2)	-2.17(2)	-1.69(8)	-3.86(8)
Belle	10.58	-4.93(2)	-6.84(2)	-4.1(1)	-10.9(1)

According to the above discussion, the results for the hadronic corrections have been obtained using the full R parameterization for the virtual corrections and the contribution of pions only for real pair emission. Moreover, we do not provide results for the hadronic corrections at the BES (center of mass) c.m. energies on top of the

Table 2: Comparison of the exact massive NNLO with BabaYaga@NLO results, in per mille.

	$\sqrt{s}$		$\sigma_{\rm BY}$ (nb)	$S_{e^+e^-}$	S_{lep}	S_{had}	S_{tot}
KLOE	1.020	NNLO		-3.935(5)	-4.472(5)	1.02(4)	-3.45(4)
		BabaYaga	455.71	-3.445(2)	-4.001(2)	0.876(5)	-3.126(5)
BES	3.097	NNLO		-2.246(8)	-2.771(8)	–	–
		BabaYaga	158.23	-2.019(3)	-2.548(3)	–	–
BES	3.650	NNLO		-1.469(9)	-1.913(9)	–1.3(1)	-3.2(1)
		BabaYaga	116.41	-1.521(4)	-1.971(4)	-1.071(4)	-3.042(5)
BES	3.686	NNLO		-1.435(8)	-1.873(8)	–	–
		BabaYaga	114.27	-1.502(4)	-1.947(4)	–	–
BaBar	10.56	NNLO		-1.48(2)	-2.17(2)	-1.69(8)	-3.86(8)
		BabaYaga	5.195	-1.40(1)	-2.09(1)	-1.49(1)	-3.58(2)
Belle	10.58	NNLO		-4.93(2)	-6.84(2)	-4.1(1)	-10.9(1)
		BabaYaga	5.501	-4.42(1)	-6.38(1)	-3.86(1)	-10.24(2)

J/ψ and $\psi(2S)$ resonances. Actually, we observed [8] that for such energies the NNLO Bhabha cross section is dominated by the contribution of narrow resonances that can not be treated like mere perturbative effects and require a separate study.

Among the corrections induced by the different particle species, it can be seen from Table 1 that the dominant contribution is given by the electron pairs, with an increasing importance of muons and hadrons at the B factories, the τ contribution being always negligible at meson factories.

In particular, among the real emission processes we concluded that only the reaction $e^+e^- \to e^+e^-e^+e^-$ gives significant contributions to the cross section used in the luminosity measurements. When the accuracy of the experiment reaches the per mille level, this process has to be considered and its contributions added to the theoretical cross section or alternatively subtracted as a background from the experimental cross section.

The main conclusion of the exact NNLO calculation is that the total correction is about 0.3 – 0.4% at KLOE, BES and BaBar and reaches about 1% at Belle. Therefore, complete NNLO massive corrections, or a their approximation, are certainly needed for precise luminosity calculations.

2.3. The NNLO massive corrections in the code BabaYaga@NLO

BabaYaga@NLO is one of the most precise and widely used theoretical tools to monitor the luminosity of meson factories. It is a QED Parton Shower generator, whose intrinsic leading logarithmic accuracy is improved by the inclusion of exact NLO soft+virtual and hard photon correction factors. In BabaYaga@NLO these NLO factors are dressed by self-energy insertions, so that a subset of the complete NNLO massive corrections previosuly discussed is included in the code. More precisely, the contributions accounted for are: *i)* the factorizable loop-by-loop corrections within the full class of NNLO virtual corrections *ii)* the contribution of real (soft and hard) photon emission in the sector of real corrections. In other words, in the generator any contribution from real pair radiation and irreducible virtual corrections is neglected, which makes the approach theoretically consistent because it avoids an imbalance of L^3 contributions.

Both leptonic and hadronic self-energy contributions are taken into account in the code. For the R parameterization we use in the present study the same VP_HLMNT_v2_0 routine as in the exact calculation, in order to perform consistent comparisons.

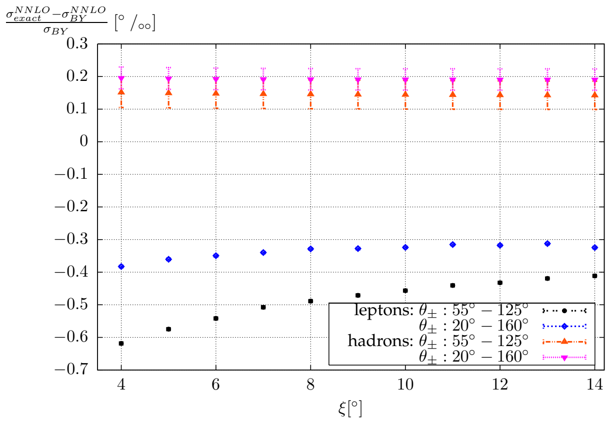
3. EXACT NNLO MASSIVE CORRECTIONS VS. BABAYAGA@NLO

The quality of the approximation inherent in BabaYaga@NLO is shown in Table 2 in comparison with the results of the exact calculation. Again realistic reference event selection cuts are considered.

It can be seen from Table 2 that there is a generally good agreement for all the meson factories experiments, the relative differences always being below the one per mille. There is a maximum registered difference of about 0.07% at Belle, where the corrections have the largest impact, but the agreement is at the level of a few units in 10^{-4} for all the other experiments. Therefore the very bulk of NNLO massive corrections is accounted for in BabaYaga@NLO.

In our analysis, we also studied how the above conclusions are stable against variation of the experimental

Figure 1: The relative difference, in per mille, of the NNLO massive leptonic and hadronic corrections between exact σ^{NNLO}_{exact} and BABAYAGA@NLO σ^{NNLO}_{BY}, as a function of an acollinearity cut for two different angular acceptance regions for KLOE-like event selections.



cuts, such as acceptance and acollinearity cuts. We noticed that for the leptonic corrections there is a slight increase in the difference for particularly tight acollinearity cuts but in general the difference is not particularly sensitive to the variation of the event selection. In any case, also for particularly severe cuts, the maximum observed difference still remains below the one per mille [8]. An example of such a cut-dependence study is shown in Fig. 1 for KLOE, which is the experiment with the smallest uncertainty in the measurement of the luminosity, at the level of 0.2%. It can be seen that a variation of the acollinearity cut from its reference value of 9° to smaller values only slightly affects the relative difference between the exact calculation and the BabaYaga@NLO approximation for leptonic correction, leaving the situation for hadrons practically unchanged.

4. CONCLUSIONS

We provided an exact calculation of the NNLO massive corrections to Bhabha scattering in QED, exploiting a number of analytical results and computational tools developed over the years. We presented exact numerical predictions for leptonic and hadronic corrections in the presence of realistic cuts for luminosity monitoring at meson factories. We compared the exact results with the approximate ones provided by the MC code BabaYaga@NLO. We concluded that NNLO massive corrections are well accounted for in the generator at an accuracy level below the one per mille. This reinforces the estimate of the total theoretical precision of BabaYaga@NLO, previously established on the grounds of only partial results for NNLO massive corrections. The present total accuracy of BabaYaga@NLO can be estimated 0.1% at KLOE and BES (excluding data taking on top of narrow resonances J/ψ and $\psi(2S)$), and about 0.15% at the B factories.

This precision is today sufficient by comparison with the experimental accuracy of meson factories. Nonetheless, there is still room for a further error reduction along the following directions. There is a need for an assessment of the theoretical accuracy for luminosity measurements in a close vicinity of narrow resonances. This requires detailed studies of the uncertainties associated to the hadronic vacuum polarisation and NNLO hadronic corrections, including beam spread effects. The leading part of the presently missing NNLO massive corrections in BabaYaga@NLO could be implemented through *e.g.* QED Structure Functions, to account for the interplay between virtual irreducible corrections and lepton pair radiation. Last but not least, the recently obtained exact results for the one-loop corrections to radiative Bhabha scattering [25], as well as for $e^+e^- \to \mu^+\mu^-\gamma$ [25, 26], are the final NNLO benchmark that should be deeper investigated in comparison with the present MC approximation.

All these perspectives are left to future works.

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